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ORIGINAL ARTICLE

# Optimal time scheduling for carrying out minor maintenance on a steam turbine

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**Abstract** Almost all the major equipment of ships or industry today undergo regular maintenance on its components and sub-components. In most of the cases it is observed that there are generally two kinds of maintenance in terms of times to repair and its effect on the availability of the main equipment; minor maintenance and a major maintenance or overhaul. The minor maintenance are generally carried at a component or a sub-component level, either at fixed intervals or based on the component condition. The minor maintenance however, requires that the equipment is brought down from operation for a shorter duration and put back in operation immediately after completion of the minor maintenance. The major maintenance on the other hand is carried out preferably in a workshop, where the equipment is completely stripped down to undertake maintenance which otherwise would not have been possible in its original location (for e.g. inside a ship). The time required for carrying out major maintenance is therefore substantial when compared to the minor maintenance period. The paper puts forward a method for scheduling minor maintenance of a major equipment—a steam turbine of a ship, within the time frame of the major maintenance interval based upon the deterioration of the components. The wear processes of the important components of the turbine are assumed to be time variant gamma processes and allow for continuous monitoring of the wear. Since turbines are a complex mix of components, sub-components and other ancillary systems, most of which are easily replaceable in miniscule of time periods, the paper considers only the most important wearing components which play a direct part in deterioration of performance of the turbine. All other random failures, which based on the past experience are only a very few in numbers and have very little impact on the availability of the turbine, have been ignored. MLE and a Gibbs sampling method has been used to estimate the parameters of distribution of gamma wear processes for the components of turbine.

**Keywords** Gamma process · Maximum likelihood · Gibbs sampling

## 1 Introduction

Mathematical optimization models are being increasingly applied today in the field of maintenance management in order to lower the cost of maintenance and failure. Some of the applications of these optimization models can be found at Dekker and Scarf (1998), Dekker (1996), Gertsbakh (1977), Frank et al. (1995), and Barbera et al. (1996). However, in most of the cases the focus of the study is to optimize the major maintenance period of the whole (read single) equipment. A wide number of minor maintenance work that is undertaken within the operational time of the main equipment is generally ignored. When the interruptions to the operation of the main equipment caused by such minor maintenance becomes a cause of worry, there comes a need to optimize the scheduling of such maintenance so that the availability of the main equipment is maximized and the cost of the operation and maintenance is minimized.

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For a warship where availability of full power from its steam turbines during crucial operation time is of paramount importance, optimization of minor maintenance period means maximum availability of power within a given period with the added advantage of having the maintenance actions planned well in advance to arrange for logistics required for the forthcoming maintenance actions.

Furthermore, the main equipment, such as the main propulsion engines of a ship may suffer from a problem of lack of data on failures of its components. However, what most of these equipment operators or maintainers would certainly have are the records of wear or deterioration for example wear recordings of its bearings, or loss of power per kg of steam flow over a period of time etc. It is this data that can be effectively utilized for optimizing the minor maintenance on the main equipment. Most of the maintenance models available in literature make the maintenance decisions against the backdrop of uncertainty in time to failure. However when it comes to deterioration or ageing, these models may not be able to model the different stages of deterioration as these are mostly adept in distinguishing the equipment as operational or failed (Van Noortwijk 2007). In order to represent deterioration on the basis of lifetime distributions, the failure rate function can be applied. However, failure rates can generally be ascertained in cases where we have a large number of sample components. For a case where we have only time based recording of deterioration or say wear of a single component, the lifetime distributions are clearly unsuitable for much use.

In the case of a steam turbine, it is opined that the deterioration must be modeled in terms of a time dependent stochastic process say $W(t)$, $t \geq 0$ where $W(t)$ represents wear (or deterioration) and is a random quantity for all $t \geq 0$. This is due to the fact that wear is an ongoing process in these machines and changes in stress, load, age and wear of the components vary the deterioration with respect to time. Though much work has been done in the past in applying gamma processes to deterioration of civil structures (Cinlar et al. 1977; Lawless and Crowder 2004; Frangpool et al. 2004; Nicolai et al. (2007)), this author did not find literature on application of gamma process applied to study the wear of mechanical components except for (Wang et al. 2000) where the gamma process was used to model the hazard rate of water pumps.

### 1.1 The steam turbine

A steam turbine is an important source of power generation onboard steam propelled ships. Super heated steam generated in a boiler is fed to a turbine at higher energy levels, in terms of its pressure and temperature, where it expands and converts its energy into mechanical energy. This mechanical energy may then be used for propelling the ship or for electrical power generation as the case may be. The turbine requires some routine minor maintenance on few parts of the turbine to ensure its efficient functioning. The author was tasked to decide the optimal time for carrying out minor maintenance on a turbine at the least operation and maintenance cost.

A cross section of a steam turbine is shown in the Fig. 1. The turbine depends on various other ancillary systems for its proper functioning. These systems are: lubricating oil system, sea water cooling system, superheated steam system, etc. All the systems are external to the turbine and would consist of pumps, filters, pipelines valves etc. Enough redundancies are built in the system to ensure that these systems have a very high degree of availability and from the experience of the operators; it is only seldom that the turbine had to be put out of operation for any maintenance on these ancillary systems. The turbine on the other hand has other sub-components which deteriorate and have to be routinely maintained so as to keep the performance of the turbine to a reasonable acceptable level. There are also few other components which can only be maintained or replaced during a major overhaul by a complete disassembly of the turbine.

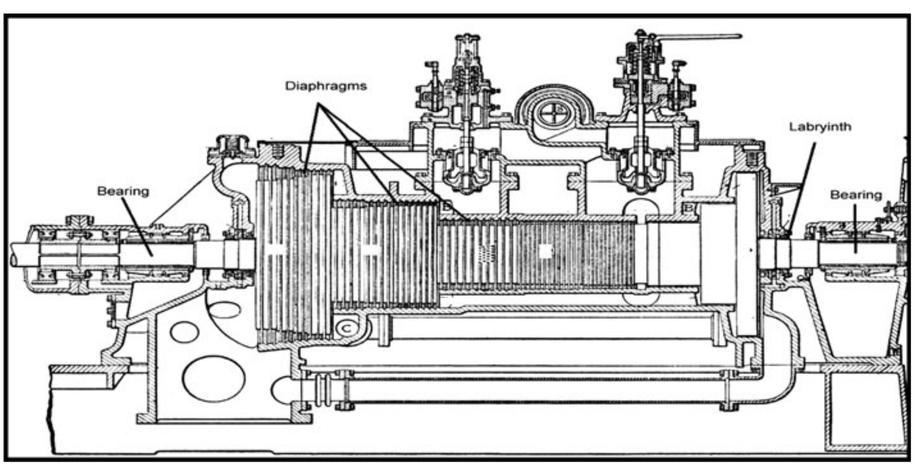


**Fig. 1** A typical steam turbine

### 1.2 Understanding the problem

The turbine operation is being continuously monitored during its operation by a watchkeeper, who records the parameters of operation viz lub oil pressure, temperature etc. and is trained to take emergency measures and avoid catastrophic failure of the turbine. From their past experiences the operators could only recall two instances (out of 12 such turbines on different ships) where the turbine had to be shut down momentarily because of a problem with the ancillary system. Here too, the available redundancies ensured that the turbines were put back in action in just a few minutes. The FTA carried out for the turbine including its ancillary systems takes on a mammoth size and would prohibit its analysis through a normal software program. This coupled with the fact that there is no need to carry out maintenance on the redundancies of the ancillary systems by bringing down the turbine meant that we can safely ignore the ancillary systems and focus only on the parts of the turbine which require minor maintenance by shutting down the turbine. This may not be a big compromise considering the fact that the components which result in failure of the turbine so as to cause a major maintenance of the turbine will play its role in altering the time frame (life time) within which we want to optimize the minor maintenance schedules.

The maintenance engineers have recognized four sets of main components and assemblies which they say deteriorate during operation resulting in an overall deterioration of the turbine performance. These components are plain bearings, diaphragm assembly (including turbine blades and other flow path components), labyrinths and a cam and nozzle (control) assembly. Out of these only three components, require regular maintenance; the bearings, cam and nozzle assembly and the labyrinths. The diaphragm assembly cannot be touched unless we strip open the turbine, which can only be done during a major maintenance period called “overhaul”. The problem is the deterioration in all the cases cannot be directly measured. For example, the deterioration or wear in diaphragm assembly cannot be directly measured in terms of clearances increased between the turbine rotors and the diaphragms. Similarly, the deterioration or wear in labyrinths which leads to discomfort by raising the humidity level in the engine room can only be measured when the labyrinths have been removed from the turbine. The wear in the bearing however, can be measured directly using a poker gauge.

### 1.3 Why gamma distribution

Gamma distribution is known as the most suitable distribution to model the monotonically increasing wear or deterioration. Here the system failure behavior might be described by e.g, a damage accumulation model Abdel Hameed M (1975; 1987) (for a mechanical system) or the evolution of a defective product rate (for a production line), or a corrosion/erosion level (for a structure). The system state at any time ‘t’ can be summarized by a random ageing variable/deterioration $W_t$. In the absence of repair or replacement actions, $W_t$ is an increasing stochastic process, with $W_0 = 0$. The system will fail when the ageing variable or deterioration exceeds a predetermine level $W_f$. The gamma process is also a reasonable extension of a deterioration process with exponential jumps. The gamma process is parameterized by $\alpha$ and $\beta$ which can be estimated from the deterioration data. If $W_t$ (deteriorating state) is a gamma process then for all $0 \leq s < t$ the random variable $W_t - W_s$ (increments of deterioration between s and t) has a gamma pdf with shape parameter $\alpha\,(t - s)$ and a scale parameter $\beta$, given by:

$$f_{\alpha(t-s),\beta}\,(w) = \frac{\beta^{\alpha\,(t-s)}}{\Gamma(\alpha(t-s))}\, w^{\alpha(t-s)-1} e^{-w\beta} I_{\{x \geq 0\}} \qquad (1)$$

The gamma process has a non-negative independent increment property. The mean and variance of its degradation rate can be expressed as $\alpha/\beta$ and $\alpha/\beta^2$. For such a process the deteriorating state starting from $w_0$, the associated failure time distribution, CDF for a given failure threshold, $W_f$ can be expressed as

$$F_{\alpha,\beta}(w) = 1 - \frac{1}{\Gamma(\alpha t)} \int_0^{(W_f - w_0)\beta} e^{-u} u^{(\alpha t-1)} du \qquad (2)$$

Wear or deterioration of any component is something that is accumulative gradual process rising monotonically over a period of time. Examples of wear or deterioration can be found in bearings, corrosion or erosion wear of pump casings and impellers etc. One thing that binds them all is that the wear or deterioration always keeps increasing, it can never decrease. But the most important motivation for choosing the gamma process to model the deterioration of the components of the turbine for this paper stems from the fact that there is no record available of failure time of the bearings or components available from other ships. What is available however is the wear data recorded over a period of inspections for the bearings. For some other components wear needs to be construed indirectly in terms of power loss. For the labyrinths of the steam turbine wear is mainly measured physically in terms of clearances, when it is being replaced. Hence what we have over the time axis is the wear data of different sets of labyrinths each of which is being renewed. The wear data of the four main wearing/deteriorating components is as given below:

| Bearings | | Labyrinths | | Diaphragm | | Cam/Nozzle assy | |
|---|---|---|---|---|---|---|---|
| Time | Wear | Time | Wear | Time | Wear | Time | Wear |
| 0.45 | 1.77 | 0.1 | 0.00001* | 0.45 | 1.11 | 0.5 | 2.75 |
| 1 | 2.44 | 0.5 | 6.889 | 1 | 2.22 | 0.9 | 7.75 |
| 1.52 | 3.422 | 0.7 | 7.33 | 1.52 | 4.44 | 1.1 | 8.75 |
| 1.8 | 3.98 | 0.75 | 7.44** | 1.8 | 4.45 | 1.4 | 9 |
| 2.85 | 5.16 | 1.2 | 8.6 | 2.85 | 5.56 | 1.6 | 9.125 |
| 3.4 | 6 | 1.4 | 8.8667 | 3.4 | 6.67 | | |
| 3.65 | 5.97*** | | | – | – | | |
| 3.9 | 6.9 | | | 3.9 | 7.8 | | |
| 4.2 | 7.3 | | | 4.2 | 8.89 | | |
| 4.5 | 8.07 | | | 4.5 | 10 | | |

*The wear could not be measured, but an arbitrary small value has been mentioned to facilitate calculation of the distribution parameters using MLE method

**Wear data are each for different sets of labyrinths (except the first two serials). The 3rd serial was actually 7.111

***Data rejected as wear could not have reduced

### 1.4 Recording of wear

The wear recordings for bearing and labyrinth are actually in dimensions of 'thous' or one thousandth of an inch—a commonly used parameter to measure clearances. However, the same has been converted into a scale of 1–12 non-dimensional parameter (12 being the extreme level) to maintain parity in scales of deterioration between different components of the turbine. The wear on the diaphragm assembly of the turbine are difficult to measure, as the clearances in these can only be measured after disassembly of the turbine (done only during major overhaul). Since wear in the diaphragms results in direct loss of power in the turbine, assuming negligible steam loss from labyrinths and pipe joints, (compared to loss through diaphragms) we record power take off per kg steam flow and scale the deterioration in it on a scale of 1–12.

## 2 Estimating the parameters of distribution

The wear is assumed to be following a time variant gamma wear process where the shape parameter is assumed to follow a power law $\alpha$ (t) = $\lambda t^{\zeta}$, the scale parameter is $\beta$ (Van Noortwijk 2007). If we take the time to inspection as $t_0$, $t_1$, $t_2 \ldots t_n$ and the wear recordings in the time intervals as $dw_1$, $dw_2$, $dw_3 \ldots . dw_n$ we can express the MLE expression of the Gamma distribution as follows

$$L = \prod_{i=1}^{i=n} \frac{\beta^{\lambda\left(t_i^{\zeta}-t_i^{\zeta}-1\right)}}{\Gamma\left(\lambda\left[t_i^{\zeta}-t_i^{\zeta}-1\right]\right)} dw_i^{\lambda\left(t_i^{\zeta}-t_i^{\zeta}-1\right)^{-1}} e^{-\beta dw_i} \text{ where}$$

$$dw_i = \text{wear between intervals} \tag{3}$$

Differentiating the logarithm of Eq. 3 we can work out the equations for the parameters of the gamma equations for the respective components as shown below. The details of the evaluation of the equations has been given at Appendix

$$\lambda t_n = W_n \beta \qquad \text{where } W_n = \sum_{i=1}^{i=n} dw_i \tag{4}$$

$$\sum_{i=1}^{n} \lambda \log \beta \left[\log t_i t_i^{\zeta} - \log t_{i-1} t_{i-1}^{\zeta}\right] - \frac{\partial\left(\log \Gamma(\lambda(t_i^{\zeta} - t_{i-1}^{\zeta}\right)}{\partial\left(\lambda\left(t_i^{\zeta} - t_{i-1}^{\zeta}\right)\right)}$$
$$\lambda\left[\log t_i t_i^{\zeta} - \log t_{i-1} t_{i-1}^{\zeta}\right]$$
$$+ \lambda \log dw_i \left[\log t_i t_i^{\zeta} - \log t_{i-1} t_{i-1}^{\zeta}\right] = 0 \tag{5}$$

$$t_n^{\zeta} \log \beta = \sum_{i=1}^{n} \left(t_i^{\zeta} - t_{i-1}^{\zeta}\right) \left[\frac{\log\left(\Gamma\left(\lambda\left(t_i^{\zeta} - t_{i-1}^{\zeta}\right)\right)\right)}{\partial\left[\lambda\left(t_i^{\zeta} - t_{i-1}^{\zeta}\right)\right]} - \log dw_i\right] \tag{6}$$

The equations above have been solved for each component using the large scale algorithm in MATLAB. The algorithm is a subspace trust region method and is based on the interior-reflective Newton method described in Coleman and Li (1996) and Coleman and Li (1994). Once the parameters have been evaluated, a Kolmogorov–Smirnov test has been carried out to arrive at the significance levels. A variance–covariance matrix has also been evaluated for the parameters as shown in Table 1. It may be noted that in the Table 1 the actual value of wear of labyrinth at t value of 0.75 is 7.111. This is because it is the value of a new labyrinth set and the values of wear of an large population at time 0.75 can be assumed to be normally distributed and therefore it is possible that the value of one set of such labyrinth can be lower than one used previously. However, the MLE method being employed to calculate the parameters fails to take up a value which is decreasing with respect to time. The modified value shown in the table as 7.444 was therefore interpolated from a previous value. The parameters evaluated this way for labyrinth were however not very encouraging as can be seen in the table below in terms of significance level and variances.

Since no other data were available for labyrinth, use was made of the Bayesian estimate method using Gibbs Sampling method (using WINBUGS program). The model created for carrying out the analysis assumed a non-informative prior for $\zeta$ whereas $\lambda$ and $\beta$ are assumed to be

**Table 1** Gamma parametric values estimated through MLE method

| Component | Parameter | Estimated value | $p$ value | (KS test) test stats | Variance–covariance matrix |
|---|---|---|---|---|---|
| Bearing | $\lambda$ | 16.2727 | 0.98 | 0.111 | [0.932144115e-2, 0.4170028691e-2, 0.1599007123e-3] |
| | $\beta$ | 6.6547 | | | |
| | $\zeta$ | 0.7975 | | | [0.4170028691e-2, 2.109578064, 0.1202680562e-1] |
| | | | | | [0.159900712e-3, 0.1202680562e-1, 0.5342011525e-2] |
| Diaphragm | $\lambda$ | 2.1386 | 0.6030 | 0.33 | [0.366218377e-3, 0.10039189e-3, −0.126069663e-3] |
| | $\beta$ | 0.7793 | | | [0.1003918954e-3, 0.2030550622, 0.189260913e-3] |
| | $\zeta$ | 0.9981 | | | [−0.126069663e-3, 0.189260913e-3, 0.89698268e-3] |
| Cam/Nozzle | $\lambda$ | 19.5844 | 0.3090 | 0.6 | [0.420403722e-2, 0.701349085e-3, 0.64909551e-4] |
| | $\beta$ | 2.9655 | | | [0.7013490855e-3, 0.3707955431, 0.26430327e-2] |
| | $\zeta$ | 0.5911 | | | [0.64909551e-4, 0.264303273e-2, 0.263898758e-2] |
| Labriynth | $\lambda$ | 4.051 | 0.0766 | 0.667 | [0.240725118e-2, 0.529587701e-3, 0.64060818e-4] |
| | $\beta$ | 0.4873 | | | [0.529587701e-3, 0.156973895, 0.66101831e-2] |
| | $\zeta$ | 1.369 | | | [0.640608183e-4, 0.66101831e-2, 0.177074e-1] |

exponentially and gamma distributed. The resulting estimates are shown in Fig. 2.

The above estimates lead to a better $p$ value as shown in Table 2 below:

Assumptions—we make the following assumptions

- That the wear of the four major components of turbine : bearings, diaphragms assembly, labyrinth and cam and nozzle assembly are the only ones that affect the performance of the turbine and all other parts have negligible impact on the turbine.
- The deterioration of the four major components are independent of each other.
- The inspections and maintenance are perfect.
- Time taken to repair is negligible when compared with the life time of the diaphragm assembly and therefore the repair can be considered to be instantaneous.
- There is a definite cost involved per degree of wear per unit time for each component

## 3 Optimization of the minor maintenance interval

We have four major components or assemblies of the turbine which deteriorate. Out of these only three components can be maintained without any major dis-assembly of the turbine. The diaphragm assembly can only be maintained when the turbine is being overhauled which is a major job and can be undertaken only during a major repair period. The question is what should be the optimal time interval to carry out the minor maintenance of the three wearing components namely bearings cam and nozzle assembly and the labyrinth assembly? If we maintain these three

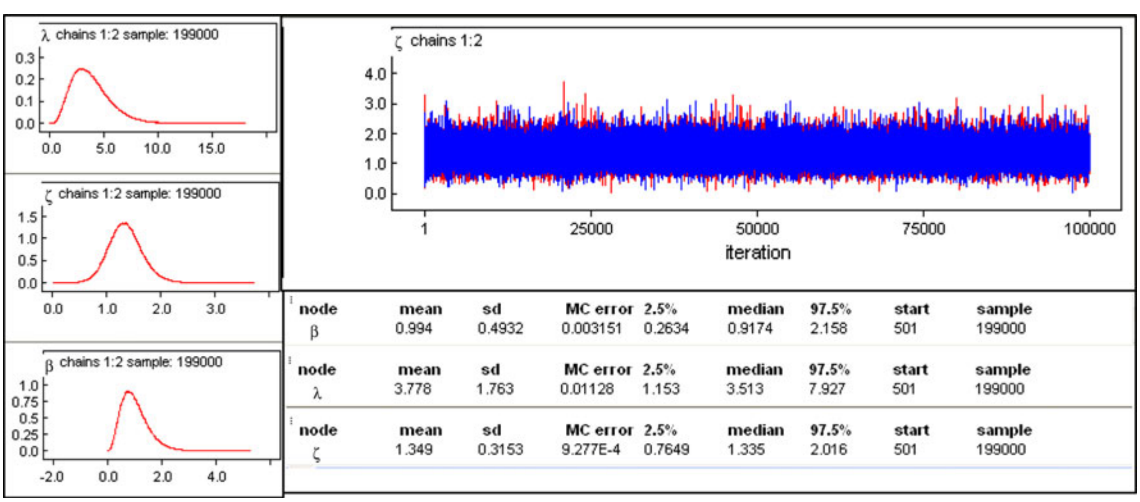


**Fig. 2** Estimates through Gibbs Sampling

**Table 2** Gamma parametric values estimated through Gibbs sampling method

| Component | Parameter | Estimated value | $p$ value | (KS test) Test stats | Variance – Covariance matrix |
|---|---|---|---|---|---|
| Labyrinth | $\lambda$ | 3.778 | 0.3180 | 0.5 | [0.50783141e-3, 0.693588689e-4, 0.5044540233e-4] |
| | $\beta$ | 0.994 | | | |
| | $\zeta$ | 1.349 | | | [0.693588689e-4, 0.370307031e-1, 0.250876398e-3] |
| | | | | | [0.50445402e-4, 0.25087639e-3, 0.1493070837e-2] |

components after each of them independently reach its tolerance limits, we will have a larger number of minor maintenance breaks. This is shown in the Fig. 3 below for deterioration limits of 10 for the bearings labyrinth and the cam and nozzle assembly where 5 maintenance actions need to be taken if done independently. However, as the result of the paper will show, with an optimal time interval for the combined minor maintenance actions, only one maintenance action will be required to be taken at an interval of 2.28 years approximately (Fig. 8).

It is intuitionally clear that instead of having multiple minor maintenance schedules for maintenance of turbine components such as labyrinths and nozzle assembly, it would be beneficial if we could find out a common schedule that is a best compromise between the maintenance schedules of the three components. It is therefore important to arrive at, if feasible, the optimal common time interval of maintenance when all the components that can be maintained without major overhaul of the turbine should be maintained within the time frame of the major overhaul period dictated by the component (in this case the diaphragm assembly) which requires major stripping down of the turbine at the workshop.

The first step is to find out the wear limits of each individual sub-component and then scale them say from 1 to 12 to maintain parity between them. It would be beneficial therefore to represent the optimization equations as a function of wear limits. This is one more reason why gamma distribution is useful for analysis of wear or deterioration of mechanical components. It can use time and deterioration interchangeably for its analysis. To demonstrate this, let us assume that the initial oil clearances in the bearing of the turbine is $W_0$ at time $t_0$ and the maximum clearance permitted is $W_f$. Then if $W_f$ is reached at time $T_f$, and W(t) is a gamma distributed deterioration at time t, we have the cumulative distribution function F at time t as:

$$F(t) = \Pr\left(T_f \le t\right) = \Pr\left(W(t) \ge \left(W_f - W_0\right)\right) = \int_{W_f - W_0}^{\infty} f(w)dw$$

If the shape parameter is a function of time $\lambda(t) = \alpha t^{\zeta}$ and the scale parameter is $\beta$ then

$$F(t) = 1 - \frac{1}{\Gamma(\alpha t^{\zeta})} \int_{0}^{Wf - W_0} e^{-u} u^{\left(\alpha t^{\zeta} - 1\right)} du \tag{7}$$

The pdf can be given as f(t)

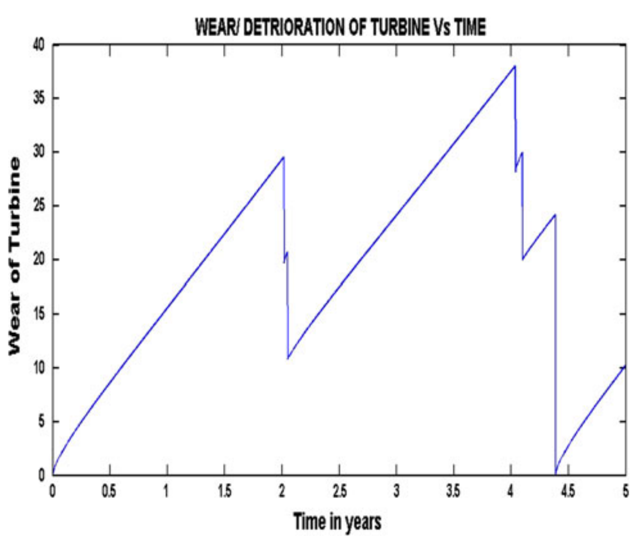


**Fig. 3** Deterioration levels with maintenance

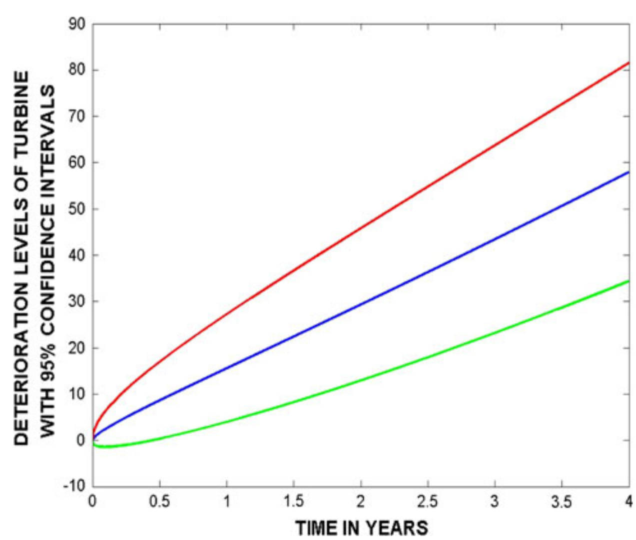


**Fig. 4** Deterioration levels without maintenance

$$= \frac{\mathrm{dF(t)}}{\mathrm{dt}}$$
$$= \frac{\zeta t^{\zeta-1}}{\Gamma(\alpha t^{\zeta})} \int\limits_{(W_f-W_0)\beta}^{\infty} \left(\log(u) - \mathrm{digam}\left(\alpha t^{\zeta}\right)\right) u^{\alpha t^{\zeta}-1} e^{-u} du \tag{8}$$

where digam is the digamma functionon the otherhand the pdf of the wear (a gamma process) can be given as

$$f_{X(t)}(x) = \frac{\beta^{\alpha t^{\zeta}}}{\Gamma(\alpha t^{\zeta})} x^{\alpha t^{\zeta}-1} e^{-\beta x} = Ga\,(x|\lambda(t), \beta), \tag{9}$$

and if the wear limit is $(W_f - W_0)$ the cdf for the accumlated wear is 1–7 i.e.

$$\mathrm{F}_{\mathrm{X(t)}}(x) = \int\limits_{0}^{(W_f-W_0)} \frac{\beta^{\alpha t^{\zeta}}}{\Gamma(\alpha t^{\zeta})} x^{\alpha t^{\zeta}-1} e^{-\beta x} dx \tag{10}$$

It may be noted that in the pdf and the cdf for accumulated wear Eqs. 9 and 10 the time ‘t’ is implicit. Also in the life time distribution pdf and cdf, the wear limit at which we consider the component to have failed is implicit.

Now, if we combine the process of deterioration of labyrinth, bearing and cam and nozzle assembly to be a one single deterioration following a gamma process, we can optimize the maintenance interval and increase the availability of the turbine within a time horizon permitted by the deterioration level of the diaphragms. We therefore assume that $f_{y1}(w)$ represents the pdf of wear of labyrinth, bearing and cam-nozzle assembly and $f_{y2}(w)$ represents the pdf of wear of diaphragm assembly. The pdfs can then be denoted by the following equations(PG Moschopoulos, “ The distribution of the sum of independent gamma random variables”, Ann Institute Statistic Mathemeatics 37 1985):

$$f_{y1\text{ or }2}(w) = \frac{C\sum_{k=0}^{\infty} \delta_k w^{\rho+k-1} e^{-w\beta_{\max}} \beta_{\max}^{\rho+k}}{\Gamma(\rho+k)} \tag{11}$$

where $\rho = \sum_{i=1}^{2} \lambda_i t^{\zeta_i}$ $i$ from 1 to 2 are components

$$\beta_{\max} = \max(\beta_i)$$

$$C = \prod_{i=1}^{2} (\beta_i/\beta_{\max})^{\lambda_i t^{\zeta i}}$$

$$\gamma_k = \sum_{i=1}^{2} \left(\lambda_i t^{\zeta_i}\right)(1 - \beta_i/\beta_{\max})^k/k,\ k = 1, 2, \ldots$$

$$\delta_{k+1} = \frac{1}{k+1} \sum_{i=1}^{k+1} i\gamma_i \delta_{k+1-i},\ k = 1, 2, \ldots$$

However, it may not be feasible to use the above expression to use it interchangeably for average wear and average time to reach a given wear. It is therefore necessary to find a convoluted pdf for combined wear of components that can be maintained during minor maintenance periods. The procedure for the processes can be described using the flow chart given in Fig. 7. The procedure relies on using an appropriate Gaussian function to map the Laplace of individual wear pdfs of individual component and then finding the CDF for the combined wear of components by inversing the Laplace using the Jagerman method (Jagerman 1995). Once the CDF function for accumulated wear is obtained, we use Eq. 10 reproduced below to arrive at the specific value for each wear limit for a particular time value ‘t’ (Fig. 5). Repeating this step a number of times for various values of time ‘t’ gives us the “reliability shape curve shown below for a chosen value of wear limit for the combined wear value of components that can be maintained during minor maintenance

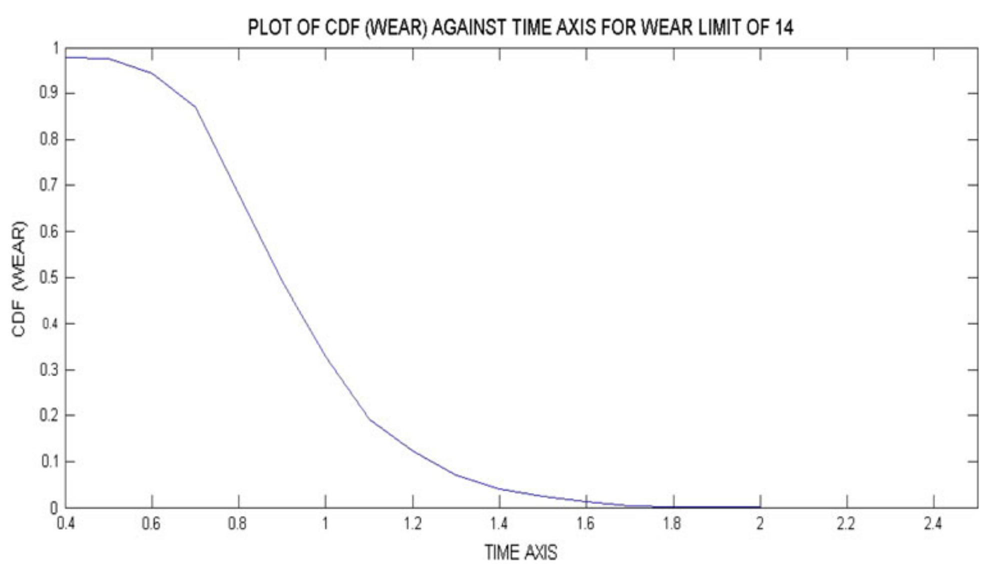


**Fig. 5** CDF values of wear for a chosen wear limit plotted against time

$$F_{X(t)}(x) = \int_{0}^{(W_f - W_0)} \frac{\beta \alpha t^{\zeta}}{\Gamma(\alpha t^{\zeta})} x^{\alpha t^{\zeta}-1} e^{-\beta x} dx$$

Repeating the above steps for various values of wear limits will then give us the Fig. 6. Please note that since the Laplace transform for the convolution has been obtained using Mont-Carlo simulation and the CDFs of wear has been calculated for a particular combined wear limit value at discrete time points (for the below example at every $t = 0.05$ years) the curves seen are not very smooth.

Mapping the above reliability curves to an appropriate Gaussian fucntion again will give us a function in value of 'x' (the wear) say R(x). Integrating this Gaussian function R(x) between 0 and infinity gives us the mean time to reach the specific combined wear limit which is required to ascertain the number of maintenance actions that will be required to be undertaken on the components of the turbine within the time frame of the major overhaul period estimated by the time taken by the diaphragm assembly (which cannot be maintained during the minor maintenance period) (Fig. 7). We can therefore represent the optimality equation as follows:

$$\min \left[\text{Cost of Operation till overhaul}\right]_{W_{\lim y1}} = C_{DN} n + C_a n \left(W_{\lim\ y1} - W_0\right) \int_0^{\infty} R(x) dx \quad (12)$$

where n = number of maintenance actions

$$= \frac{\int_0^{\infty} \frac{1}{\Gamma(\alpha t^{\zeta})} \int_0^{W_{\lim\ y2} - W_0} e^{-u} u^{\left(\alpha t^{\zeta} - 1\right)} du dt}{\int_0^{\infty} R(x)\, dx} \quad (13)$$

$C_{DN}$ = cost of minor maintenance actions + cost of unavailibility of turbine

$C_a$ = cost rate of running turbine per degree of wear per unit time;

$W_0$ = wear at time 0

$W_{limy(i)}$ = limiting wear of components at time; $i = 1$ – bearing, cam & labyrinth; $i = 2$ – diaphragm assy

$$R\ (x) = \text{Gaussian function of form '}\ a_1 e^{\left(-((x-b_1)/c_1)^2\right)} + a_2 e^{\left(-((x-b_2)/c_2)^2\right)} + a_3 e^{\left(-((x-b_3)/c_3)^2\right)}, \quad (14)$$

that maps the curves for individual wear limits shown at Fig. 6

## 4 Results

Using the steps given in the flow chart above and selecting the values of wear limits as 10 for all the components ($W_{limy}$), cost of maintenance actions and unavailability of 67,000 (averaged out for all the wearing components) and cost of running components in worn condition per degree wear per unit time as 3,200/(degwear * time in years) we evaluate that the optimal time interval for carrying out the minor maintenance on the bearings labyrinth and the cam and nozzle assembly is 2.28 years at the optimum combined wear limit of 25 as seen in the graph below. This is because the cost due to the maintenance actions is the dominant cost in the above problem and hence at the mean

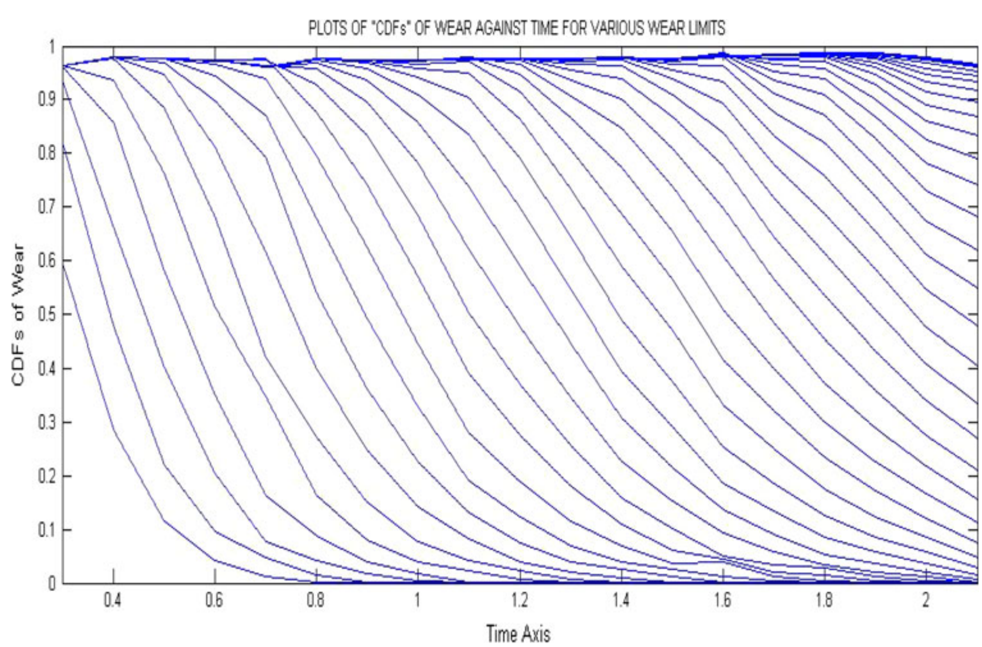


**Fig. 6** CDF values of wear for various wear limits plotted against time

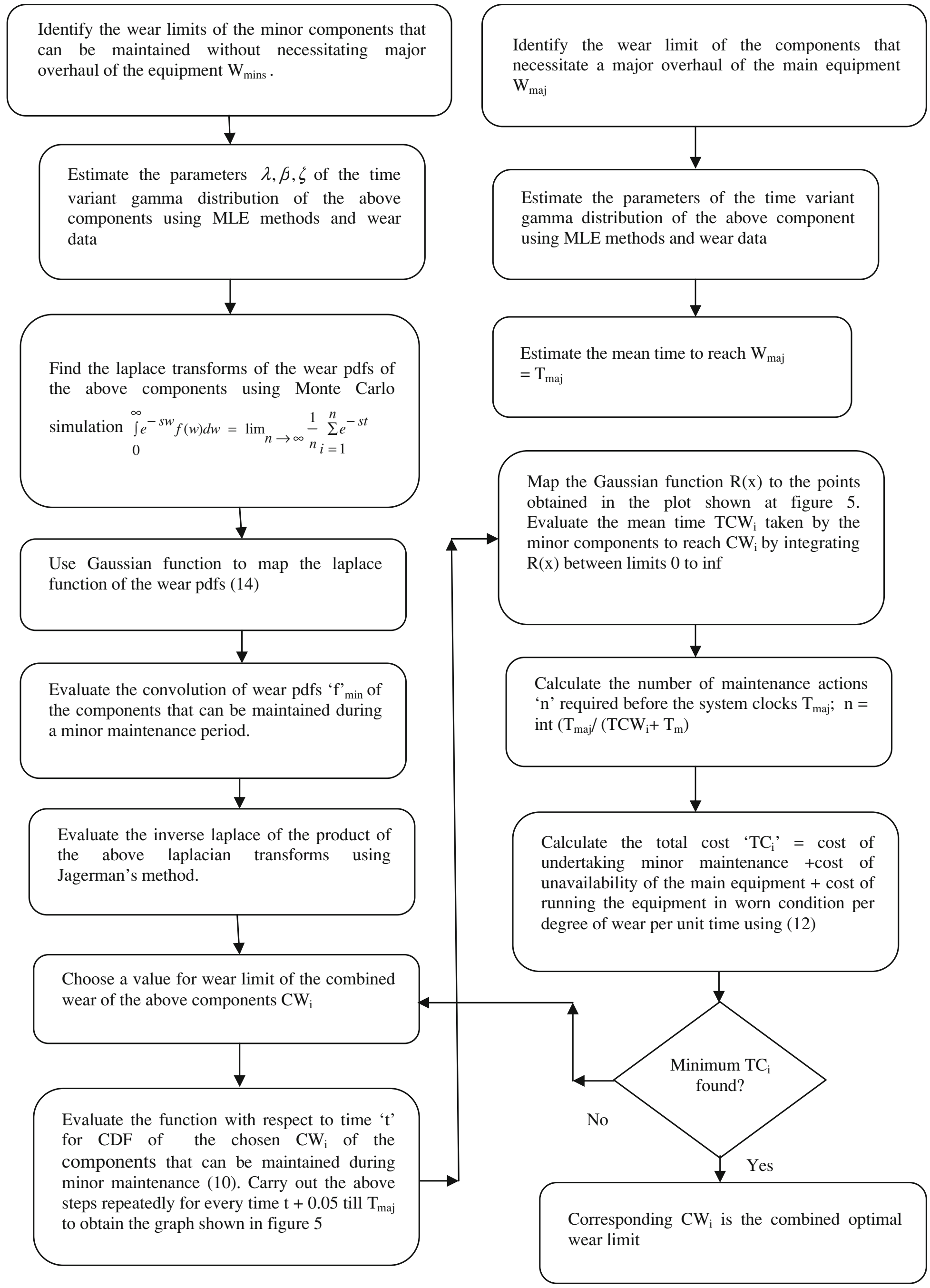


**Fig. 7** Flow chart for procedure to evaluate the optimal time interval for carrying out minor maintenance

time of 2.28 years (1/2 of the major overhaul period of 4.56 years) only one minor maintenance action should be undertaken when all the components such as bearings, labyrinths and the cam and nozzle assembly will get maintained. The next minor maintenance action will fall due together with the major overhaul period (Fig. 8).

Further, the probability that the wear of any of the grouped component may cross its individual wear limit before the combined wear limit reaches its combined wear limit (25) can be assessed using equation R(x) from (14). For example the probability that the component cam and nozzle assembly will cross its individual limit of 10 at time $t = 2.28$ years is 0.6378 (using Eqs. 1–10) however the probability that this component would cross its limit and the combined wear limit would remain within the 25.0 optimal limit is 0.6378 * 0.3632 = 0.2317. Similarly we have for other components

$$P_{cam} = 0.2317;\ P_{bearing} = 2.1901e - 007;\ P_{labryinth} = 0.2368$$

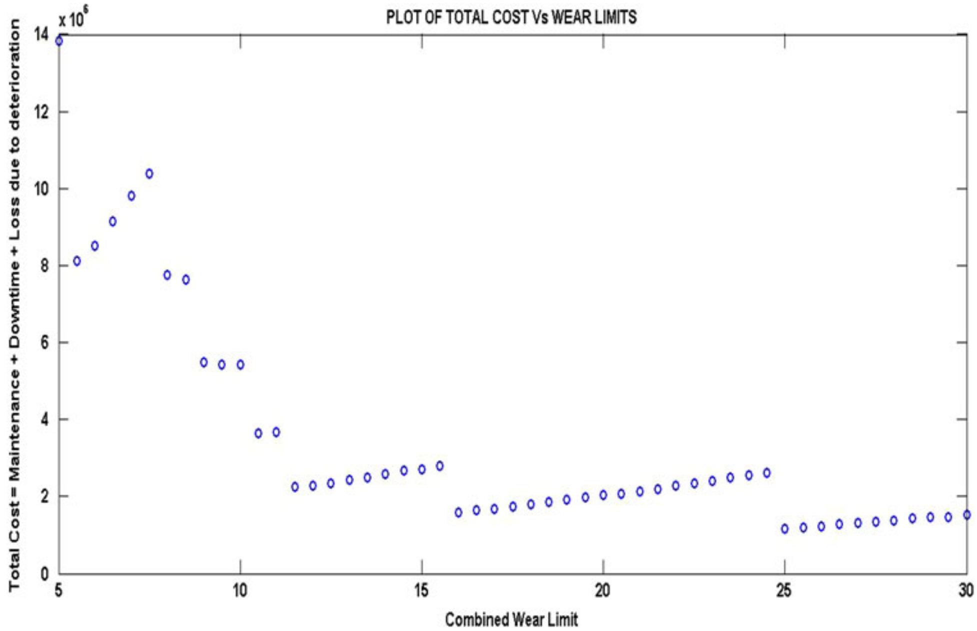


**Fig. 8** Plot of total cost vs Combined wear limits of Labyrinth, bearing and cam assembly

It is important to note that though there is a probability of labyrinth and cam nozzle assemblies exceeding their individual wear limits of 12 without the combined wear limit reaching the 25.0 mark, the only penalty being paid is that in terms of steam leakage which increases the discomfort of the operators and power loss. The bearings which might result in significant problems if its exceed its limit is however a remote possibility.

Further, with the property of the time variant gamma wear process to increase in variance with increase in time (Fig. 4), one may make use of the process by predicting the immediate next maintenance period more accurately than an estimate of maintenance somewhere distant in future. To use the knowledge of the wear measured by the maintainer at time say $t_1$ the maintainer can predict the further course of wear more accurately, but for this we would need to convert the above gamma process into a time stationary wear process. We therefore replace $t^{\zeta}$ *by* $\tau$ and calculate the expected deterioration at a later time $\tau_2$ $(\tau_2 > \tau_1)$ considering $\tau_1 = 0$ and wear $w_1 = 0$ at $\tau_1 = 0$.

As an example consider the bearing wear readings at Table 1. If wear w = 0 at time t = 0; then the expected wear at t = 1.52 is w = 3.4147 with a variance of 0.7163. The formula used for expected deterioration is as given below

$$E(w) = \int_{0}^{\infty} w \frac{\beta^{\lambda t^{\zeta}}}{\Gamma(\lambda t^{\zeta})} w^{\lambda t^{\zeta}-1} e^{-w\beta} I_{\{x \geq 0\}} \tag{15}$$

However on converting the gamma process into a stationary gamma process by replacing $t^{\zeta}$ *by* $\tau$, and considering the wear w = 2.4453 as 0 at $\tau = 0$ we get the same expected total wear as w = 3.4147 with a variance of 0.3817

## 5 Conclusion

The paper puts forward a method to optimize the maintenance intervals of multiple minor maintenance actions using deterioration data available with the maintainers of the equipment. Gamma distributions have been used to model the wear/deterioration of the components since they cater to the temporal variability and are useful when records of failure of components are not available and only the deterioration or wear data are available. Another advantage of this process is that it allows time and deterioration to be used interchangeably for analysis and hence optimization of maintenance decisions can be based on the deteriorating condition of the components of the concerned equipment. Cases where there is strict problem with overshooting of the individual wear limits of the components, optimization of interval for a combined maintenance action can be progressed within the constraint of the wear limit of the specific component.

## Appendix

Maximum likelihood function for gamma variables

Let there be "n" observations of the subject component whose parameters of gamma wear are to be known. Let these observations be carried out at different times $t_1$,

$t_2 \ldots t_n$, then if the shape parameter is function of time such that $\alpha = \lambda t^{\zeta}$ and scale parameter is $\beta$; then probability of observation of exactly these "n" values of variables can be given by

$$L = \prod_{i=1}^{i=n} \frac{\beta^{\lambda\left(t_i^{\zeta}-t_i^{\zeta}-1\right)}}{\Gamma\left(\lambda\left[t_i^{\zeta}-t_i^{\zeta}-1\right]\right)} dw_i^{\lambda\left(t_i^{\zeta}-t_i^{\zeta}-1\right)^{-1}} e^{-\beta dw_i} \text{ where } dw_i = \text{wear between intervals}$$

Taking logarithm of maximum likelihood we have

$$\text{Log (L)} = \sum_{i=1}^{i=n} \frac{\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log(\beta)}{\log\Gamma\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)} + \left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)-1\right)\log(\delta_i) - \beta dw_i$$

Taking partial derivatives for wrt $\beta$, $\lambda$ and $\zeta$ we will have

$$\begin{aligned}
\frac{\partial(\log L)}{\partial\beta} &= 0 = \sum_{i=1}^{n} \lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\frac{1}{\beta} - dw_i \therefore \lambda t_n \\
&= W_n\beta \frac{\partial(\log L)}{\partial\lambda} = 0 = \sum_{i=1}^{n}\left(t_i^{b}-t_{i-1}^{b}\right)\log\beta \\
&- \frac{\log\left(\Gamma\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)\right)}{\partial\lambda} + \left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log dw_i \\
&= \sum_{i=1}^{n}\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log\beta \\
&- \frac{\log(\Gamma\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)}{\partial\left[\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right]} \frac{\partial\left[\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right]}{\partial\lambda} \\
&+ \left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log dw_i = \sum_{i=1}^{n}\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log\beta \\
&- \frac{\log\left(\Gamma\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)\right)}{\partial\left[\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right]} \\
&\times \left[t_i^{\zeta}-t_{i-1}^{\zeta}\right] + \left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log dw_i
\end{aligned} \tag{16}$$

$$\begin{aligned}
&\therefore \sum_{i=1}^{n}\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\left[\log dw_i - \frac{\log\left(\Gamma\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)\right)}{\partial\left[\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right]}\right] \\
&= -\sum_{i=1}^{n}\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\log\beta \therefore t_n^{\zeta}\log\beta \\
&= \sum_{i=1}^{n}\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\left[\frac{\log(\left(\Gamma\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)\right)}{\partial\left[\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right]} - \log dw_i\right]
\end{aligned} \tag{17}$$

$$\begin{aligned}
\frac{\partial(\log L)}{\partial\zeta} &= \sum_{i=1}^{n}\lambda\log\beta\left[\log t_i t_i^{\zeta} - \log t_{i-1}t_{i-1}^{\zeta}\right] \\
&- \frac{\partial\left(\log\Gamma(\lambda(t_i^{\zeta}-t_{i-1}^{\zeta}\right)}{\partial\zeta} \\
&+ \lambda\log dw_i\left[\log t_i t_i^{\zeta} - \log t_{i-1}t_{i-1}^{\zeta}\right] \\
&= \sum_{i=1}^{n}\lambda\log\beta\left[\log t_i t_i^{\zeta} - \log t_{i-1}t_{i-1}^{\zeta}\right] \\
&- \frac{\partial\left(\log\Gamma(\lambda(t_i^{\zeta}-t_{i-1}^{\zeta}\right)}{\partial\left(\lambda\left(t_i^{\zeta}-t_{i-1}^{\zeta}\right)\right)}\lambda\left[\log t_i t_i^{\zeta} - \log t_{i-1}t_{i-1}^{\zeta}\right] \\
&+ \lambda\log dw_i\left[\log t_i t_i^{\zeta} - \log t_{i-1}t_{i-1}^{\zeta}\right] l
\end{aligned} \tag{18}$$

$\therefore$ where $\frac{\partial(\log\Gamma(A)}{\partial(A)}$ in above equations is a digamma function Nicolai et al. (2007).

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